Orbifolds versus smooth heterotic compactifications

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Abstract. Following the recent exploration of smooth heterotic compactifications with unitary bundles, orbifold compactifications in six dimensions can be shown to correspond in the blow-up to compactifications with U(1) gauge backgrounds. A powerful tool is the comparison of anomaly polynomials. The presentation here focuses on heterotic $SO(32)$ compactifications in six dimensions including five-branes. Four dimensional and $E_8 \times E_8$ models are briefly commented on.

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1 Introduction

Orbifold compactifications of the heterotic string on the one hand have been employed for the past twenty years, and on the other hand heterotic compactifications with $SU(n)$ ($n = 4, 5$) bundles on smooth manifolds have developed in the last ten years. These two seemingly unrelated approaches are shown to be closely related when the orbifold blow-ups are interpreted as smooth compactifications with $U(1)$ bundles instead of the $SU(n)$ bundles which have been the main focus for model building. For T^4/Z_N embeddings with $N = 2, 3$, there is a direct identification of the orbifold shift vector with the embedding of a line bundle L [1],

$$\frac{1}{N}(1_{n_1}, 2_{n_2}, \ldots, 0_{n_0}) \rightarrow (L_{n_1}, L^2_{n_2}, \ldots, 0_{n_0}), \qquad (1)$$

where the lower index n_i denotes the number of identical entries i in the shift vector and $\sum_i n_i = 16$. Furthermore, compactifications with $U(n)$ bundles amplify the possibilities to obtain four dimensional vacua with standard model or GUT gauge groups and are S-dual to Type I compactifications with non-Abelian bundles on D9-branes.

The present exposition is for concreteness focused on $SO(32)$ heterotic compactifications in six dimensions. Four dimensional cases and $E_8 \times E_8$ compactifications are briefly commented on.

2 Heterotic T^4/Z_N Orbifolds

Abelian T^{2n}/Z_N orbifolds of the heterotic string are described by two shift vectors, the space-time shift $\boldsymbol{v}$ which encodes a Z_N rotation $z_j \rightarrow e^{2\pi i v_j} z_j$ on $j = 1 \ldots n$ complex coordinates, and a gauge shift $\boldsymbol{V}$ which embeds the orbifold action in the gauge degrees of freedom. The vectorial shift vectors for n complex compact dimensions and embeddings in $SO(32)$ are given by

$$\boldsymbol{v} = \frac{1}{N}(\sigma_1 \ldots \sigma_n), \quad \boldsymbol{V} = \frac{1}{N}(\Sigma_1 \ldots \Sigma_{16}), \qquad (2)$$

with σ_j, Σ_k integer and $\sum_j \sigma_j = 0$ to ensure that the space-time orbifold is the singular limit of a Calabi-Yau n-fold. In order to obtain supersymmetric models, two stringy constraints have to be met. These are on the one hand the quadratic 'level-matching' condition, which ensures the modular invariance of the partition function and mixes space-time and gauge shifts, and on the other hand a linear condition on the gauge shift ensuring the existence of spinors in the gauge bundle,

$$N \sum_i \left(V_i^2 - v_i^2\right) = 0 \bmod 2, \quad N \sum_i V_i = 0 \bmod 2. \qquad (3)$$

The massless spectrum for a given choice of shift vectors in $SO(32)$ embeddings is obtained as follows: those $SO(32)$ weight vectors $\boldsymbol{w} = (\underline{\pm 1, \pm 1, 0_{14}})$ with $\boldsymbol{w} \cdot \boldsymbol{V} \in Z$ provide the non-Abelian generators of the gauge group and those with $\boldsymbol{w} \cdot \boldsymbol{V} \notin Z$ the untwisted matter states. The total rank of the gauge group is 16, and depending on the chosen gauge shift the gauge group can contain several $U(1)$ factors.

The twisted spectrum consists of $n = 1 \ldots N - 1$ sectors with twisted ground states obtained from $\boldsymbol{w} - n\boldsymbol{V}$. Oscillators lift the tachyionic vacuum to the massless level, and multiplicities are obtained from the counting of space-time fixed points. For the T^4/Z_N orbifolds with $\boldsymbol{v} = \frac{1}{N}(1, -1)$ these are 16 Z_2 fixed points for $N = 2$, nine Z_3 fixed points for $N = 3$ or four Z_4 and 16 Z_2 fixed points for $N = 4$ and one Z_6, nine Z_3 and 16 Z_2 fixed points for $N = 6$. As an example, the spectra for the 'standard embeddings' with $\boldsymbol{V} = \frac{1}{N}(1, 1, 0^{14})$ are listed in table 1.

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The gauge group is $SO(28) \times SU(2)^2$ for $N = 2$ and $SO(28) \times SU(2) \times U(1)$ otherwise. A counting of non-Abelian degrees of freedom yields $10(\mathbf{28}, \mathbf{2}) + 66(\mathbf{1})$ for all N with identical $U(1)$ charge assignments in the untwisted sectors, but different $U(1)$ charges $1 + \frac{m}{n}$ in the n^{th} twisted sector and m integer.

Table 1. Spectra of $SO(32)$ T^4/Z_N standard embeddings.

	N=2	N=3
θ^0	$(\mathbf{28}, \mathbf{2}, \mathbf{2}) + 4(\mathbf{1})$	$(\mathbf{28}, \mathbf{2})_1 + 2(\mathbf{1})_0 + (\mathbf{1})_2$
θ^1	$8(\mathbf{28}, \mathbf{1}, \mathbf{2}) + 32(\mathbf{1}, \mathbf{2}, \mathbf{1})$	$\left\{\begin{array}{c} 9(\mathbf{28}, \mathbf{2})_{\frac{1}{3}} + 45(\mathbf{1})_{\frac{2}{3}} \\ +18(\mathbf{1})_{\frac{4}{3}} \end{array}\right.$
	N=4	N=6
θ^0	$(\mathbf{28}, \mathbf{2})_1 + 2(\mathbf{1})_0$	$(\mathbf{28}, \mathbf{2})_1 + 2(\mathbf{1})_0$
θ^1	$\left\{\begin{array}{c} 4(\mathbf{28}, \mathbf{2})_{\frac{1}{2}} + 24(\mathbf{1})_{\frac{1}{2}} \\ +8(\mathbf{1})_{\frac{3}{2}} \end{array}\right.$	$(\mathbf{28}, \mathbf{2})_{\frac{2}{3}} + 8(\mathbf{1})_{\frac{1}{3}} + 2(\mathbf{1})_{\frac{5}{3}}$
θ^2	$5(\mathbf{28}, \mathbf{2})_0 + 32(\mathbf{1})_1$	$\left\{\begin{array}{c} 5(\mathbf{28}, \mathbf{2})_{\frac{1}{3}} + 22(\mathbf{1})_{\frac{2}{3}} \\ +10(\mathbf{1})_{\frac{4}{3}} \end{array}\right.$
θ^3	-	$3(\mathbf{28}, \mathbf{2})_0 + 22(\mathbf{1})_1$

For any given six dimensional spectrum, field theory anomalies arising from fermions and tensors running in loops can be computed. Using the complete list of T^4/Z_N spectra for $N = 2, 3, 4$ and some $N = 6$ examples, the anomaly polynomial for $SO(32)$ heterotic orbifold compactifications takes the form [1]

$$\begin{aligned} I_8 = \big(&\mathrm{tr}R^2 + \sum_i \alpha_i \mathrm{tr}_{SO(2M_i)} F^2 + \sum_j \beta_j \mathrm{tr}_{SU(N_j)} F^2 \\ &+ \sum_k \gamma_k F^2_{U(1)_k} + \sum_{i<j} \delta_{ij}\, F_{U(1)_i}\, F_{U(1)_j}\big) \times \big(\mathrm{tr}R^2 - \\ &- \sum_i \mathrm{tr}_{SO(2M_i)} F^2 - 2 \sum_j \mathrm{tr}_{SU(N_j)} F^2 + \sum_k \tilde{\gamma}_k F^2_{U(1)_k}\big) \end{aligned} \tag{4}$$

with $\alpha_0 = 2$ for all $SO(2M_0)$ gauge groups with fundamental representations only, $\alpha_1 \leq 1$ otherwise, $\beta_j \leq 2$ and nearly always even, $\gamma_k < -7$ and $\tilde{\gamma}_k \leq -2$ with the latter always even. The T^4/Z_3 orbifold admits at most one $SO(2M_0)$ gauge factor associated with M_0 zero entries in the gauge shift $\boldsymbol{V}$ and $\alpha_0 = 2$, whereas the T^4/Z_N orbifolds with N even admit a second $SO(2M_1)$ gauge factor associated with the entries $\varSigma_{i_1} = \ldots = \varSigma_{i_{M_1}} = N/2$ with spinorial representations in the twisted spectrum and $\alpha_1 \leq 1$. The anomaly polynomial at the orbifold point factorises completely into 4×4, and as discussed below for the smooth $K3$ compactifications, this signals the fact that $U(1)$ factors at the orbifold point are massless.

The scalar potential in six dimensions is completely determined by D-term interactions,

$$V = \sum_{a,\alpha} D^{a,\alpha} D^{a,\alpha} \quad \text{with} \quad D^{a,\alpha} = \varPhi_i^\dagger \sigma^a t^\alpha_{ij} \varPhi_j, \tag{5}$$

with the Pauli matrices σ^a, generators of the gauge groups t^α_{ij} and matter fields $\varPhi_j$.

The anomaly polynomials and gauge shift vectors admit by comparison an interpretation of the orbifold models as smooth compactifications with $U(1)$ bundles as discussed in the next section provided that twisted scalars receive vacuum expectation values along flat directions of the scalar potential (5) thereby blowing up the singularities and breaking the orbifold point $U(1)$.

3 The heterotic string on K3

Smooth compactifications of the heterotic string on Calabi-Yau n-folds require for the gauge bundle $\bar{F} = \oplus_i \bar{F}_i$ to preserve supersymmetry that each component $\bar{F}_i$ is a holomorphic (1,1)-form satisfying at tree level the primitivity condition $\int_{CY_n} J^{n-1} \wedge \mathrm{tr}\bar{F}_i = 0$. On $K3 = CY_2$ the latter condition is exact, whereas on CY_3 the primitivity condition receives a 1-loop correction. Furthermore, consistent compactifications require the Bianchi identity on the 3-form $H = dB - \frac{\alpha'}{4}(\omega_{YM} - \omega_L)$ which is quadratic in the gauge bundle to be satisfied as well as the K-theory constraint which is linear in the bundle,

$$\mathrm{tr}\bar{F}^2 - \mathrm{tr}\bar{R}^2 = 0, \quad (2\pi)^{-1}\mathrm{tr}\bar{F} \in H^2(CY_n, 2Z). \tag{6}$$

Eq. (6) can be directly compared to its orbifold counterpart (3). The massless spectrum is obtained by decomposing the adjoint representation of $SO(32)$,

$$\mathbf{496} \rightarrow \left(\begin{array}{c} (\mathbf{Anti}_{SO(2M)}) \\ \sum_j (\mathbf{Adj}_{U(N_j)}; \mathbf{Adj}_{U(n_j)}) \\ \sum_j (\mathbf{Anti}_{U(N_j)}; \mathbf{Sym}_{U(n_j)}) + c.c \\ \sum_j (\mathbf{Sym}_{U(N_j)}; \mathbf{Anti}_{U(n_j)}) + c.c. \\ \sum_{i<j} (\mathbf{N}_i, \mathbf{N}_j; \mathbf{n}_i, \mathbf{n}_j) + (\mathbf{N}_i, \bar{\mathbf{N}}_j, \mathbf{n}_i, \bar{\mathbf{n}}_j) + c.c. \\ \sum_j (\mathbf{2M}, \mathbf{N}_j; \mathbf{n}_j) + c.c. \end{array}\right), \tag{7}$$

and embedding $U(n_j)$ bundles V_j inside $U(N_j n_j)$ factors. The gauge group is $SO(2M) \times \prod_j U(N_j)$ with the massless representations counted by cohomology classes of the associated bundles as listed in table 2. The chiral part of the spectrum is computed from the

Table 2. Massless spectra in terms of cohomology classes.

reps.	$SO(2M) \times \prod_i U(N_i)$
$(\mathbf{Adj}_{U(N_i)})_{0(i)}$	$H^*(CY_n, V_i \otimes V_i^*)$
$(\mathbf{Sym}_{U(N_i)})_{2(i)}$	$H^*(CY_n, \bigwedge^2 V_i)$
$(\mathbf{Anti}_{U(N_i)})_{2(i)}$	$H^*(CY_n, \bigotimes^2_s V_i)$
$(\mathbf{N}_i, \mathbf{N}_j)_{1(i),1(j)}$	$H^*(CY_n, V_i \otimes V_j)$
$(\mathbf{N}_i, \bar{\mathbf{N}}_j)_{1(i),-1(j)}$	$H^*(CY_n, V_i \otimes V_j^*)$
$(\mathbf{Adj}_{SO(2M)})_0$	$H^*(CY_n, \mathcal{O})$
$(\mathbf{2M}, \mathbf{N}_i)_{1(i)}$	$H^*(CY_n, V_i)$

Euler characters of the bundles W,

$$\chi(CY_n, W) = \sum_{j=0}^{n} (-1)^j \mathrm{dim} H^j(CY_n, W)$$

$$= \int_{CY_n} \mathrm{ch}(W)\mathrm{Td}(CY_n), \qquad (8)$$

with the Chern characters $\mathrm{ch}_k(W) = (k!(2\pi)^k)^{-1}\mathrm{tr}\bar{F}^k$ of the bundles and the Todd class of the manifold $\mathrm{Td}(CY_n) = 1 + \frac{1}{12}c_2(CY_n) + \ldots$. For $K3 = CY_2$, the index $\chi(K3, W) = \mathrm{ch}_2(W) + 2\,\mathrm{rank}(W)$ actually counts the number of vector minus the number of hyper multiplets, and the complete massless spectrum is easily computed for a given embedding.

The $U(1)$s in smooth compactifications generically become massive through the generalised Green-Schwarz mechanism involving antisymmetric tensor modes on the CY_n, namely in terms of the expansion of the ten dimensional dual 6-form $B^{(6)} = \ell_s^{2n-2} b_k^{(8-2n)} \sum_k \widehat{\omega}_k + \ldots$ along $(2n-2)$ forms $\widehat{\omega}_k$ on CY_n, the mass terms arising from wrapped antisymmetric tensor modes are given by

$$S_{mass} \sim \ell_s^{2n-8} \sum_k \sum_i \int_{M^{10-2n}} b_k^{(8-2n)} \wedge [\mathrm{tr} F_i \bar{F}_i]^k, \qquad (9)$$

i.e. the $U(1)$ masses are of the order of the string scale. For $K3$ compactifications, this is a complete set of mass terms, whereas on CY_3-folds, also the coupling to the unwrapped antisymmetric tensor mode contributes.

As an example, consider a line bundle L embedded in $U(2)$, also denoted as $(L, L, 0_{14})$, with $\mathrm{ch}_2(L) = -12$ in order to fulfill the Bianchi identity. The gauge group is $SO(28) \times SU(2)_{\times U(1)_{massive}}$ with matter spectrum $10(\mathbf{28}, \mathbf{2})_1 + 46(\mathbf{1}, \mathbf{1})_2$ and twenty neutral hyper multiplets parameterising the $K3$ geometry. The counting of non-Abelian representations agrees with that of the orbifold 'standard embeddings' in section 2.

For the most general embedding of $U(n_i)$ bundles in $SO(32)$, the anomaly polynomial in six dimensions is given by [2]

$$\begin{aligned} I_8 = \frac{1}{3}\big(\sum_i c_1(V_i)\mathrm{tr}_{U(N_i)}F\big) \times \\ \times \big(\sum_j c_1(V_j)\big[\mathrm{tr}_{U(N_j)}F\,\mathrm{tr}R^2 - 16\mathrm{tr}_{U(N_j)}F^3\big]\big) \\ + \big(\mathrm{tr}R^2 + 2\mathrm{tr}_{SO(2M)}F^2 + 4\sum_i(\mathrm{ch}_2(V_i) + n_i)\mathrm{tr}_{U(N_i)}F^2\big) \\ \times \big(\mathrm{tr}R^2 - \mathrm{tr}_{SO(2M)}F^2 - 2\sum_i n_i\,\mathrm{tr}_{U(N_i)}F^2\big). \end{aligned} \qquad (10)$$

The anomaly eight-form (10) on $K3$ factorises as $2 \times 6 + 4 \times 4$, where the '2' arises from those Green-Schwarz couplings (9) providing $U(1)$ masses.

Comparing the coefficients of the non-Abelian gauge factors in the last two lines of (10) with those at the orbifold point (4) leads to $\alpha_i \stackrel{!}{=} 2$, $\beta_j \stackrel{!}{=} 4(\mathrm{ch}_2(V_j) + n_j)$, $-2 \stackrel{!}{=} -2n_j$. The last condition, $n_j = 1$, reveals that orbifold models correspond in the blown-up phase to smooth embeddings with $U(1)$ bundles. The smooth instanton numbers $\mathrm{ch}_2(V_j)$ can then be computed using the second identification, and finally $\alpha_0 = 2$ is fulfilled for those $SO(2M_0)$ gauge factors at the orbifold point arising from zeros in the shift vector. In the other cases, the smooth gauge group is only $SU(M_1) \subset SO(2M_1)$, and spinorial representations decompose into singlets and antisymmetric representations of $SU(M_1)$.

The primitivity condition $\int_{CY_n} J^{n-1} \wedge \mathrm{tr}\bar{F}_i = 0$ at tree level is at the orbifold point trivially fulfilled since the gauge bundle is localised at fixed points whose exceptional divisors have zero volume. More results on the blowing-up procedure are given in [3,4].

4 Including five-branes

It is straightforward to include some non-perturbative objects, the heterotic 5-branes. For $SO(32)$ compactifications, N_a coincident 5-branes provide a $Sp(2N_a)$ gauge factor, and the matter spectrum in table 2 is extended by antisymmetric and bifundamental states counted by extensions rather than cohomologies as listed in table 3. The sky-scraper sheafs $\mathcal{O}|_a$ describing the 5-branes have $\mathrm{ch}(\mathcal{O}|_a) = (0, 0, -\gamma_a, 0)$ where $\gamma_a = 1$ for a 5-brane which is point like on $K3$ and γ_a the Poincaré dual 4-form of a 5-brane wrapping a 2-cycle on CY_3. The Bianchi identity is modified to

Table 3. Massless states from 5-branes counted by extension groups.

reps.	$SO(2M) \times \prod_i U(N_i) \times \prod_a Sp(2N_a)$
$(\mathbf{Anti}_{Sp(2N_a)})$	$\mathrm{Ext}^*_{CY_n}(\mathcal{O}\|_a, \mathcal{O}\|_a)$
$(\mathbf{N}_i, \mathbf{2N}_a)_{1(i)}$	$\mathrm{Ext}^*_{CY_n}(V_i, \mathcal{O}\|_a)$
$(\mathbf{2M}, \mathbf{2N}_a)$	$\mathrm{Ext}^*_{CY_n}(\mathcal{O}, \mathcal{O}\|_a)$
$(\mathbf{2N}_a, \mathbf{2N}_b)$	$\mathrm{Ext}^*_{CY_n}(\mathcal{O}\|_a, \mathcal{O}\|_b)$

$$\mathrm{tr}\bar{F}^2 - \mathrm{tr}\bar{R}^2 - 16\pi^2 N_a \gamma_a = 0, \qquad (11)$$

and the supersymmetry conditions on the bundles are unchanged. The anomaly polynomial (10) receives in the third line an additional term $-2\mathrm{tr}_{Sp(2N_a)}F^2$ [2]. On $K3$, bifundamental representations of 5-branes at different points are massive with the mass proportional to their distance. There is one hyper multiplet in the antisymmetric representation of $Sp(2N_a)$, n_i hyper multiplets transforming as $(\mathbf{N}_i, \mathbf{2N}_a)_{1(i)}$ and a half-hyper multiplet in the $(\mathbf{2M}, \mathbf{2N}_a)$ representation.

As an example, take again a line bundle L embedded in $U(2)$, but this time with $\mathrm{ch}_2(L) = -3$. The Bianchi identity is fulfilled in the presence of 18 5-branes. The gauge group is $SO(28) \times SU(2)_{\times U(1)_{massive}} \times Sp(36)$, and the matter spectrum consists of $(\mathbf{28}, \mathbf{2}_1; \mathbf{1}) + (\mathbf{1}, \mathbf{2}_1; \mathbf{36}) + \frac{1}{2}(\mathbf{28}, \mathbf{1}; \mathbf{36}) + 10(\mathbf{1}, \mathbf{1}_2; \mathbf{1}) + (\mathbf{1}, \mathbf{1}; \mathbf{630})$. In addition, there are the universally present twenty neutral hyper multiplets.

On the orbifold side, as discussed in [5] for T^4/Z_N cases, 5-branes act as magnetic sources on fixed points thereby shifting the vacuum energy in the corresponding left-moving non-supersymmetric twist sector as well as entering the modular invariance constraint while the

right-moving supersymmetric sector is not affected. 5-branes away from the fixed points provide the same matter states as in the smooth $K3$ case, but at fixed points gauge enhancements can occur, as e.g. $Sp(2N_a) \to U(2N_a)$ for the S-dual D5-branes in the Type I orbifold of [6].

In [5], the T^4/Z_3 embedding with $\boldsymbol{V} = \frac{1}{3}(1,1,0_{14})$ and 18 5-branes is listed with perturbative gauge group $SO(28) \times SU(2)_{\times U(1)}$. Omitting the $U(1)$ charges, the untwisted spectrum is identical to that of the standard embedding in table 1, $(\mathbf{28},\mathbf{2}) + 3(\mathbf{1})$, but the twisted spectrum contains only $9(\mathbf{1}) + 18(\mathbf{1}^\star)$. Adding the states charged under $Sp(36)$, the massless spectrum of the smooth example with 5-branes is recovered except for the massless $U(1)$ at the orbifold point.

5 Some results in four dimensions

The discussion of the stringy consistency conditions and generic spectrum has been presented for CY_n folds for any n. The concrete form of the index counting chiral states depends on the dimension and is for CY_3-folds given by

$$\chi(CY_3, W) = \int_{CY_3} \left(\mathrm{ch}_3(W) + \frac{1}{12} c_2(CY_3) c_1(W) \right). \tag{12}$$

Furthermore, as anticipated above, the primitivity condition on supersymmetric bundles receives a 1-loop correction [7]

$$\int_{CY_3} J^2 \wedge \mathrm{tr}\bar{F}_i - \frac{2\, g_s^2}{3} \int_{CY_3} \left(\mathrm{tr}\bar{F}_i^3 - \frac{1}{16} \mathrm{tr}\bar{F}_i \wedge \mathrm{tr}\bar{R}^2 \right) = 0. \tag{13}$$

This is the S-dual generalisation [9] to non-Abelian bundles on curved backgrounds of the 'MMMS' calibration condition [10] for supersymmetric D9-branes.

The holomorphicity condition is now trivially fulfilled, but the requirement that the 1-loop corrected gauge kinetic function is real gives a new constraint,

$$N_i \int_{CY_3} J^3 - 6\, g_s^2 \int_{CY_3} J \wedge \left(\mathrm{tr}\bar{F}_i^2 - \frac{N_i}{48}\, \mathrm{tr}\bar{R}^2 \right) > 0. \tag{14}$$

The mass terms from couplings to wrapped modes of the antisymmetric tensor are as given above. In addition, the coupling of the reduction of the antisymmetric tensor with two external indices, $B^{(2)} = b_0^{(2)} + \ldots$, also provides a mass coupling,

$$S^0_{mass} \sim \frac{1}{3} \sum_i \int_{M^4} b_0^{(2)} \wedge \mathrm{tr}F_i \left(\mathrm{tr}\bar{F}_i^3 - \frac{1}{16} \mathrm{tr}\bar{F}_i\, \mathrm{tr}\bar{R}^2 \right). \tag{15}$$

The $U(n)$ bundles provide new possibilities for model building with some first results reported in [7,8] for Complete Intersection and elliptically fibered Calabi Yau threefolds, respectively.

At the orbifold point, the tree level and 1-loop part of the supersymmetry condition vanish separately since the volume of the exceptional divisors is zero, but the gauge bundle and curvature have only support there.

6 Results on $E_8 \times E_8$ compactifications

The Bianchi identity, K-theory constraint and supersymmetry condition at tree level are the same as for the $SO(32)$ case presented above. In four dimensions, however, the 1-loop contribution to the supersymmetry condition differs. On the one hand, it involves all bundles inside the same E_8 factor, on the other hand, also the 5-brane positions enter. This is in contrast to the $SO(32)$ case where the supersymmetry conditions on bundles and 5-branes are decoupled. In six dimensions, $E_8 \times E_8$ 5-branes along the non-compact directions provide tensor multiplets, whereas in four dimensions, space-time filling 5-branes wrap compact 2-cycles and provide $U(1)$ gauge groups.

The mass terms from wrapped antisymmetric tensor modes have the same formal expression (9) as for the $SO(32)$ case, but in four dimensional compactifications the coupling to the universal $b_0^{(2)}$ has a different shape.

The differences between the $E_8 \times E_8$ and $SO(32)$ compactifications can be traced back to the fact that E_8 has no fourth order Casimir. Moreover, while $U(1)$ and $SU(n)$ groups arise naturally in breakings of E_8, $U(n)$ bundles can only be implemented in a very restricted way, e.g. as $U(n_1) \times U(n_2)$ with $c_1(V_{n_1}) = -c_1(V_{n_2})$.

More details about smooth six dimensional $E_8 \times E_8$ constructions on $K3$ with $U(n)$ bundles are given in [2,1], the corresponding supersymmetry conditions on CY_3 are given in [11] without and in [12] with 5-branes.

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